10/23 Lecture Notes (3.1)

No quiz this week

Ex] $T(\vec{x})=\begin{bmatrix}1&2\\0&1\\0&0\end{bmatrix}$ is 1-1 because columns are L.I.
is not onto because row of 0's in the bottom

$T(\vec{x})=\begin{bmatrix}1&-2&0\\0&1&0\end{bmatrix}$ is Not 1-1 because column of 0's
is onto because every row has a pivot so it will span $\mathbb{R}^2$

component map form? & Review Tx (Matrix??)

$T(\vec{x})=\begin{bmatrix}1&2\\0&1\end{bmatrix}$ is 1-1 because columns are L.I.
is onto because every row has a pivot

for $\mathbb{R}^n \to \mathbb{R}^m$ if # of inputs of $\mathbb{R}^n$ = # of inputs of $\mathbb{R}^n$, then being 1-1 is equiv. to being onto

$T(\vec{x})=\begin{bmatrix}0&1\\0&0\end{bmatrix}$ is Not 1-1 because column of 0's
is Not onto because row of 0's.

Note: $\begin{bmatrix}x_1\\ \vdots\\ x_m\end{bmatrix}=x_1\underbrace{\begin{bmatrix}1\\0\\ \vdots\\0\end{bmatrix}}_{\vec{e}_1}+x_2\underbrace{\begin{bmatrix}0\\1\\ \vdots\\0\end{bmatrix}}_{\vec{e}_2}+\dots+x_m\underbrace{\begin{bmatrix}0\\0\\ \vdots\\1\end{bmatrix}}_{\vec{e}_m}$

Thm] Let T be a linear transformation then there exists a Matrix A such that $T(\vec{x})=A\vec{x}$

Pf] Let $T:\mathbb{R}^m\to\mathbb{R}^n$ be a L.T. and let $T(\vec{e}_i)=\vec{a}_i$ and $A=[\vec{a}_1 \dots \vec{a}_m]$
$=([T(\vec{e}_1)\ \dots\ T(\vec{e}_m)])$ and $T\left(\begin{bmatrix}x_1\\x_2\\ \vdots\\x_m\end{bmatrix}\right)=T(x_1\vec{e}_1+\dots+x_m\vec{e}_m)$
$=x_1T(\vec{e}_1)+\dots+x_mT(\vec{e}_m)$
$=x_1\vec{a}_1+\dots+x_m\vec{a}_m=$ ∅

3.2 - Matrix Algebra

0_{nm} = $n\times m$ matrix of all 0's

Addition & Scalar Multiplication: Let A&B be $n\times m$

$T:\mathbb{R}^m\to\mathbb{R}^n$ $T(\vec{x})=A\vec{x}$
$S:\mathbb{R}^m\to\mathbb{R}^n$ $S(\vec{x})=B\vec{x}$
What is $(T+S)(\vec{x})$? $T(\vec{x})+S(\vec{x})$
Thus: $(A+B)(\vec{x})=(T+S)(\vec{x})=T(\vec{x})+S(\vec{x})$
$=A\vec{x}+B\vec{x}$

thus if $A=[\vec{a}_1,\dots,\vec{a}_m]$, $B=[\vec{b}_1,\dots,\vec{b}_m]$
$A+B=[(\vec{a}_1+\vec{b}_1)\dots(\vec{a}_m+\vec{b}_m)]$

Input columns, output rows

Similarly if c is a scalar, $cA=[c\vec{a}_1,\dots c\vec{a}_m]$ (you can only add matrices of same dimension)

Ex] $-2\begin{bmatrix}1&2\\3&4\\0&7\end{bmatrix}+3\begin{bmatrix}1&0\\1&1\\0&1\end{bmatrix}=\begin{bmatrix}1&-4\\-3&-5\\0&-11\end{bmatrix}$

For matrices A, B, E $n\times m$:

a) $A+B=B+A$
b) $c(A+B)=cA+cB$
c) $(c+d)A=cA+dA$
d) $(A+B)+E=A+(B+E)$
e) $(cd)A=$ ∅

Matrix multiplication = function composition

$A=n\times m$ Matrix, $B=r\times s$ Matrix

$T=\mathbb{R}^m\to\mathbb{R}^n$ by $T(\vec{x})=A\vec{x}$
$S=\mathbb{R}^s\to\mathbb{R}^r$ by $S(\vec{x})=B\vec{x}$
$(n\times \underbrace{m)=(r}\times s)$

so let A be $n\times m$, B be $m\times s$

let $B=[b_1,\dots,b_s]$ $x=\begin{bmatrix}x_1\\ \vdots\\x_s\end{bmatrix}$

then $S(\vec{x})=B\vec{x}=x_1\vec{b}_1+\dots+x_s\vec{b}_s$